

MARKING SCHEME MATHEMATICS CLASS 11

SECTION-A

- $\frac{(n+2)!}{(n+1)!} = \dots\dots\dots$
A) $(n+1)!$ B) $(n+2)!$ **C) $(n+2)$** D) $(n+1)$
- A square matrix $A = [a_{ij}]_{m \times n}$ is called upper triangular if:
A) $a_{ij} = 0, \forall i > j$ B) $a_{ij} = 0, \forall i < j$ C) $a_{ij} = 0, \forall i = j$ D) $a_{ij} = 1, \forall i = j$
- The concept of complex numbers as $a+bi$ form was given by.....
A) **Gauss** B) Newton C) Archimedes D) Euler
- If a square matrix A has two identical rows or columns then $\det(A) = \dots\dots\dots$
A) **Zero** B) not equal to zero C) negative D) none of these
- The period of $\sin \frac{2}{3}x$ is
A) π B) 2π **C) 3π** D) 4π
- If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, then the expression $\vec{a} \cdot (\vec{b} \cdot \vec{c})$ is
A) Scalar triple product B) Volume of parallelepiped **C) Meaningless** D) Dot product
- The axis of symmetry of the parabola $y=3x^2 - 6x+1$ is
A) $x = -1$ **B) $x = 1$** C) $x = -2$ D) $x = 2$
- The maximum value of the function $f(x,y)=2x+4y$ subjected to the constraints $x \geq 3$ and $y \geq 3$ is
A) 24 B) 20 **C) 18** D) 4
- If terminal ray of θ is in the fourth quadrant, then $\frac{\theta}{2}$ lies in quadrant.
A) First **B) Second** C) Third D) Fourth
- If $y = \sin 6\theta$ then frequency is
A) 2π B) $\frac{\pi}{3}$ **C) $\frac{3}{\pi}$** D) $\frac{2\pi}{3}$
- If A is a non zero matrix then number of non zero row in its echelon form is called..... of the matrix.
A) Solution **B) Rank** C) Value D) none of these
- The number of terms in the expansion of $(a + b)^{100}$ is
A) 99 B) 100 **C) 101** D) 102
- The sum of the odd coefficient in the binomial expansion of $(1 + x)^n$ is equal to.....
A) 2^n B) $2^{(n+1)}$ C) $2^{(n-2)}$ **D) $2^{(n-1)}$**

14. Two vectors \vec{a} and \vec{b} are parallel, for scalar λ if and only if
- A) $a \neq b$ B) $a = \lambda + b$ C) $a = \lambda b$ D) none of these
15. If $f(x) = \frac{1}{x}$ then domain of $f(x)$ is
- A) \mathcal{R} B) $R - 0$ C) $\mathcal{R} - \{0\}$ D) ∞
16. If $\sin\theta = \frac{4}{5}$, then $\sin 3\theta = \dots\dots\dots$
- A) $\frac{11}{125}$ B) $\frac{33}{125}$ C) $\frac{44}{125}$ D) $\frac{22}{125}$
17. A coin is flipped thrice. The number of sample space points are
- A) 3 B) 8 C) 9 D) 12
18. $1^2 + 2^2 + 3^2 + \dots + n^2 = \dots\dots\dots$
- A) $\frac{n}{2}$ B) $\frac{n(n+1)}{2}$ C) $\frac{n(n+1)(2n+1)}{6}$ D) $\left(\frac{n(n+1)}{2}\right)^2$
19. If none of the angle of a triangle is right angle is called triangle.
- A) Obtuse B) **Oblique** C) Acute D) None
20. Infinite geometric series is convergent if and only if
- A) $|r| < 1$ B) $|r| \geq 1$ C) $|r| > 1$ D) $|r| \geq 1$

Section-B

Q-No-1(i) Solution: $z^3 = -1$

$$z^3 + 1 = 0$$

$$z^3 + 1^3 = 0$$

$$(z + 1)(z^2 - 2z + 1) = 0$$

$$\text{Either } z + 1 = 0 \text{ or } z^2 - 2z + 1 = 0$$

$$z = -1 \text{ or } z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Or } z = \frac{+2 \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$

$$\text{Or } z = \frac{+2 \pm \sqrt{4-4}}{2(1)}$$

$$\text{Or } z = \frac{2}{2}$$

$$\text{Or } z = 1$$

Solution is $z = -1$ and $z = 1$

2 marks

2 marks

1 mark

Q-No-1(ii) Solution: Given vector $\vec{v} = \hat{i} + \sqrt{2}\hat{j} + \hat{k}$

$$\text{Now } \hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

$$\hat{v} = \frac{\hat{i} + \sqrt{2}\hat{j} + \hat{k}}{|\hat{i} + \sqrt{2}\hat{j} + \hat{k}|}$$

$$\hat{v} = \frac{\hat{i} + \sqrt{2}\hat{j} + \hat{k}}{\sqrt{1+2+1}}$$

$$\hat{v} = \frac{\hat{i} + \sqrt{2}\hat{j} + \hat{k}}{2}$$

$$\hat{v} = \frac{1}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j} + \frac{1}{2}\hat{k}$$

So direction cosines are $\cos \alpha = \frac{1}{2}$, $\cos \beta = \frac{\sqrt{2}}{2}$ and $\cos \gamma = \frac{1}{2}$

Now angle with z - axis is $\cos \gamma = \frac{1}{2}$

$$\gamma = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\gamma = 60^\circ = \frac{\pi}{3}$$

Q-No-1(iii) Solution: $L - H - S = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix} \text{ by } C_1 - C_2 \text{ and } C_2 - C_3$$

Expanding from R_1

2 marks

2 marks

1 mark

2 marks

$$= 0 \begin{vmatrix} b-c & c \\ b^3-c^3 & c^3 \end{vmatrix} - 0 \begin{vmatrix} a-b & c \\ a^3-b^3 & c^3 \end{vmatrix} + 1 \begin{vmatrix} a-b & b-c \\ a^3-b^3 & b^3-c^3 \end{vmatrix}$$

$$= 0 + 0 + 1 \begin{vmatrix} a-b & b-c \\ (a-b)(a^2+ab+b^2) & (b-c)(b^2+bc+c^2) \end{vmatrix}$$

2 marks

Taking common $a-b$ and $b-c$ from C_1 and C_2 respectively

$$= (a-b)(b-c) \begin{vmatrix} 1 & 1 \\ a^2+ab+b^2 & b^2+bc+c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(b^2+bc+c^2-a^2-ab-b^2)$$

$$= (a-b)(b-c)\{b(c-a)+c^2-a^2\}$$

$$= (a-b)(b-c)\{b(c-a)+(c+a)(c-a)\}$$

$$= (a-b)(b-c)(c-a)(a+b+c)$$

$$= R-H-S$$

1 marks

Hence $L-H-S=R-H-S$

Q-No-1(iv) Solution:

$$L-H-S = \frac{1+\tan^2\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$$

$$= \frac{1+\frac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{1-\frac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}$$

$$= \frac{\frac{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{\frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}$$

$$= \frac{1}{\cos \alpha}$$

$$= \sec \alpha$$

$$= R-H-S$$

2 marks

2 marks

1 mark

Hence $L-H-S = R-H-S$

Q-No-1(v) Solution:

Given $n(S) = 36$

Event for sum of 10 is $A = \{(4,6), (5,5), (6,4)\}$

So $n(A) = 3$

Event for sum of 11 is $B = \{(5,6), (6,5)\}$

So $n(B) = 2$

1 mark

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

2 marks

Probability that the sum of digits is 10 or 11 is

$$P(A \cup B) = P(A) + P(B) = \frac{1}{12} + \frac{1}{18}$$

$$P(A \cup B) = \frac{3+2}{36} = \frac{5}{36}$$

Now probability that the sum of digits is neither 10 nor 11 is

$$P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{5}{36} = \frac{31}{36}$$

2 marks

Q-No-1(vi) Solution:

Given x is nearly equal to 1

i.e $x = 1 + h$ where h is so small that h^2 and higher power are neglected.

Now taking

$$L - H - S = px^p - qx^q$$

$$= p(1 + h)^p - q(1 + h)^q$$

$$= p(1 + ph + \dots) - q(1 + qh + \dots) \text{ using Binomial series}$$

2 marks

Neglecting h^2 and higher powers

$$= p(1 + ph) - q(1 + qh)$$

$$= p + p^2h - q - q^2h$$

$$= (p - q) + (p^2 - q^2)h$$

$$= (p - q) + (p + q)(p - q)h$$

$$= (p - q)\{1 + (p + q)h\}$$

$$= (p - q)(1 + h)^{p+q}$$

$$= (p - q)(x)^{p+q}$$

$$= R - H - S$$

2 marks

1 mark

Hence $L - H - S = R - H - S$

Q-No-1(vii) Solution:

Let A_1, A_2, \dots, A_n are n arithmetic mean between a and b

So Arithmetic sequence is

$a, A_1, A_2, \dots, A_n, b$

Now

$$a + A_1 + A_2 + \dots + A_n + b = S_{n+2}$$

$$A_1 + A_2 + \dots + A_n + (a + b) = \frac{n+2}{2} \{2a + (n+1)d\}$$

$$A_1 + A_2 + \dots + A_n = \frac{n+2}{2} \{2a + (n+1)d\} - (a + b)$$

$$A_1 + A_2 + \dots + A_n = \frac{n+2}{2} \{a + a + (n+1)d\} - (a + b)$$

$$A_1 + A_2 + \dots + A_n = \frac{n+2}{2} \{a + b\} - (a + b) \quad \because b = a + (n+1)d$$

$$A_1 + A_2 + \dots + A_n = (a + b) \left\{ \frac{n+2}{2} - 1 \right\}$$

$$A_1 + A_2 + \dots + A_n = (a + b) \left\{ \frac{n+2-2}{2} \right\}$$

$$A_1 + A_2 + \dots + A_n = n \frac{(a+b)}{2}$$

2 marks

2 marks

1 marks

Hence proved.

Q-No-1(viii) Solution:

Given $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 3 & 2 & -1 & 0 \\ 2 & -1 & 0 & 1 \end{bmatrix}$

Now

$$\begin{array}{l} \text{R} \\ \text{R} \\ \text{R} \end{array} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 3 & 2 & -1 & 0 \\ 2 & -1 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} \text{R} \\ \text{R} \\ \text{R} \end{array} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & -4 & -1 & -9 \\ 0 & -5 & 0 & -5 \end{bmatrix} \quad \text{by } R_2 + (-3R_1) \text{ and } R_3 + (-2R_1)$$

$$\begin{array}{l} \text{R} \\ \text{R} \\ \text{R} \end{array} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & -1 & -4 \\ 0 & -5 & 0 & -5 \end{bmatrix} \quad \text{by } R_2 + (-1R_3)$$

$$\begin{array}{l} \text{R} \\ \text{R} \\ \text{R} \end{array} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & -5 & -25 \end{bmatrix} \quad \text{by } R_3 + 5R_2$$

3 marks

2 marks

So Rank of $A = 3$

1 marks

Q-No-1(ix) Solution:

$$\text{Given } f(x) = \frac{x+5}{x-6}$$

Domain of $f(x) = \mathbb{R} - \{6\}$

Now to find f^{-1}

$$f(x) = y$$

$$x = f^{-1}(y) \rightarrow (1)$$

$$\text{As } y = \frac{x+5}{x-6}$$

$$xy - 6y = x + 5 \text{ by cross multiplication}$$

$$xy - x = 6y + 5$$

$$x = \frac{6y+5}{y-1} \rightarrow (2)$$

Comparing eq(1) and eq(2)

$$f^{-1}(y) = \frac{6y+5}{y-1}$$

Replace y by x

$$f^{-1}(x) = \frac{6x+5}{x-1}$$

So Domain of $f^{-1}(x) = \mathbb{R} - \{1\}$

and Range of $f^{-1}(x) = \mathbb{R} - \{6\} \because \text{Range of } f^{-1} = \text{Domain of } f$

2 marks

2 marks

1 marks

Q-No-1(x) Solution:

$$\vec{a} = \lambda\hat{j} + 3\hat{k}, \quad \vec{b} = 2\hat{i} - \hat{j} - \hat{k} \quad \vec{c} = \hat{i} + 3\hat{j} + \hat{k}$$

Given vectors are coplanar

So

$$\vec{a} \cdot \vec{b} \times \vec{c} = 0$$

$$\begin{vmatrix} 0 & \lambda & 3 \\ 2 & -1 & -1 \\ 1 & 3 & 1 \end{vmatrix} = 0$$

Expanding from R_1

$$0 \begin{vmatrix} -1 & -1 \\ 3 & 1 \end{vmatrix} - \lambda \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 0$$

$$0 - \lambda(2 + 1) + 3(6 + 1) = 0$$

$$-3\lambda = -21$$

$$\lambda = 7$$

3 marks

2 marks

Q-No-1(xi) Solution:

$$L - H - S = 1 + \cos \beta$$

$$= 1 + \frac{a^2 + c^2 - b^2}{2ac} \quad \because \cos \beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{2ac + a^2 + c^2 - b^2}{2ac}$$

$$= \frac{(a+c)^2 - b^2}{2ac}$$

$$= \frac{(a+c+b)(a+c-b)}{2ac}$$

$$= R - H - S$$

3 marks

2 marks

Hence $L - H - S = R - H - S$

Q-No-1(xii) Solution:

Given $1 + 4x + 7x^2 + 10x^3 + \dots$

$$S_n = ?$$

As
$$S_n = \frac{a_1}{1-r} + \frac{dr}{(1-r)^2} - \frac{dr^n}{(1-r)^2} - \frac{(a_1 + (n-1)d)r^n}{1-r}$$

Here $a_1 = 1$, $d = 3$ and $r = x$

$$S_n = \frac{1}{1-x} + \frac{3x}{(1-x)^2} - \frac{3x^n}{(1-x)^2} - \frac{(1 + (n-1)3)x^n}{1-x}$$

$$S_n = \frac{1}{1-x} - \frac{(1+3n-3)x^n}{1-x} + \frac{3x}{(1-x)^2} - \frac{3x^n}{(1-x)^2}$$

$$S_n = \frac{1 - (3n-2)x^n}{1-x} + \frac{3x(1-x^{n-1})}{(1-x)^2}$$

2 marks

2 marks

1 marks

Q-No-1(xiii) Solution:

$$\text{let } \theta = \frac{\pi}{2} - \cos^{-1} x \quad \rightarrow (1)$$

$$\cos^{-1} x = \frac{\pi}{2} - \theta$$

$$x = \cos\left(\frac{\pi}{2} - \theta\right) \quad \text{for } 0 \leq \frac{\pi}{2} - \theta \leq \pi$$

$$x = \cos\left(\frac{\pi}{2} - \theta\right) \quad \text{for } -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$x = \sin \theta \quad \text{for } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

So

$$\theta = \sin^{-1} x$$

Put in eq(1)

$$\sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x$$

Hence

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

2 marks

2 marks

1 marks

Section-C

Attempt Any Three question of the following question. Each question carry equal marks.

Q.2) i) prove that for any equilateral triangle $r : R : r_1 = 1 : 2 : 3$.

Solution:

Let the measure of each side of a triangle be denoted by "c"

$$\text{Therefore } S = \frac{a+b+c}{2} = \frac{c+c+c}{2} = \frac{3c}{2}$$

$$\Delta = \sqrt{S(S-c)^3} = \sqrt{\frac{3c}{2} \left(\frac{3c}{2} - c\right)^3} = \sqrt{\frac{3c}{2} \left(\frac{c}{2}\right)^3} = \frac{\sqrt{3}c^2}{4}$$

$$\Delta = \frac{\sqrt{3}c^2}{4}$$

$$R = \frac{abc}{4\Delta} = \frac{c^3}{4 \cdot \frac{\sqrt{3}c^2}{4}} = \frac{c}{\sqrt{3}} \quad r = \frac{\Delta}{S} = \frac{\frac{\sqrt{3}c^2}{4}}{\frac{3c}{2}} = \frac{c}{2\sqrt{3}}$$

$$r = \frac{c}{2\sqrt{3}}$$

$$r_1 = \frac{\Delta}{S-a} = \frac{\frac{\sqrt{3}c^2}{4}}{\frac{3c}{2} - c} = \frac{\sqrt{3}c}{2}$$

$$r_1 = \frac{\sqrt{3}c}{2}$$

$$\text{Now } r : R : r_1 = \frac{c}{2\sqrt{3}} : \frac{c}{\sqrt{3}} : \frac{\sqrt{3}c}{2}$$

$$r : R : r_1 = \frac{c}{2\sqrt{3}} \times \frac{\sqrt{3}}{c} : \frac{c}{\sqrt{3}} \times \frac{\sqrt{3}}{c} : \frac{\sqrt{3}c}{2} \times \frac{\sqrt{3}}{c}$$

$$r : R : r_1 = \frac{1}{2} : 1 : \frac{3}{2}$$

$$r : R : r_1 = 1 : 2 : 3$$

2 marks

2 marks

1 marks

ii) Use Cramer's rule to solve $x - y + 4z = 4$, $2x + 2y - z = 2$, $3x - 2y + 3z = -3$.

Solution:

$$x - y + 4z = 4$$

$$2x + 2y - z = 2$$

$$3x - 2y + 3z = -3$$

In terms of matrices:

$$\begin{bmatrix} 1 & -1 & 4 \\ 2 & 2 & -1 \\ 3 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$$

$$\text{Where } A = \begin{bmatrix} 1 & -1 & 4 \\ 2 & 2 & -1 \\ 3 & -2 & 3 \end{bmatrix}, B = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & -1 & 4 \\ 2 & 2 & -1 \\ 3 & -2 & 3 \end{vmatrix} = 1 \begin{vmatrix} 2 & -1 \\ -2 & 3 \end{vmatrix} - (-1) \begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix} + 4 \begin{vmatrix} 2 & 2 \\ 3 & -2 \end{vmatrix} \\ &= 1(6 - 2) + 1(6 + 3) + 4(-4 - 6) \end{aligned}$$

2 marks

$$= 4 + 9 - 40$$

$$= -27$$

$$|A| = -27$$

Now for "x" using Cramer's rule

$$|A_x| = \begin{vmatrix} 4 & -1 & 4 \\ 2 & 2 & -1 \\ -3 & -2 & 3 \end{vmatrix} = 4 \begin{vmatrix} 2 & -1 \\ -2 & 3 \end{vmatrix} - (-1) \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} + 4 \begin{vmatrix} 2 & 2 \\ -3 & -2 \end{vmatrix}$$

$$= 4(6 - 2) + 1(6 - 3) + 4(-4 + 6)$$

$$= 16 + 3 + 8$$

$$= 27$$

$$|A_x| = 27$$

Now for "y" using Cramer's rule

$$|A_y| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 2 & -1 \\ 3 & -3 & 3 \end{vmatrix} = 1 \begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix} - (4) \begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix} + 4 \begin{vmatrix} 2 & 2 \\ 3 & -3 \end{vmatrix}$$

$$= 1(6 - 3) - 4(6 + 3) + 4(-6 - 6)$$

$$= 3 - 36 - 48$$

$$= -81$$

$$|A_y| = -81$$

Now for "z" using Cramer's rule

$$|A_z| = \begin{vmatrix} 1 & -1 & 4 \\ 2 & 2 & 2 \\ 3 & -2 & -3 \end{vmatrix} = 1 \begin{vmatrix} 2 & 2 \\ -2 & -3 \end{vmatrix} - (-1) \begin{vmatrix} 2 & 2 \\ 3 & -3 \end{vmatrix} + 4 \begin{vmatrix} 2 & 2 \\ 3 & -2 \end{vmatrix}$$

$$= 1(-6 + 4) + 1(-6 - 6) + 4(-4 - 6)$$

$$= -2 - 12 - 40$$

$$= -54$$

$$|A_z| = -54$$

NOW

$$x = \frac{|A_x|}{|A|} = \frac{27}{-27} = -1$$

$$y = \frac{|A_y|}{|A|} = \frac{-81}{-27} = 3$$

$$z = \frac{|A_z|}{|A|} = \frac{-54}{-27} = 2$$

So $(x, y, z) = (-1, 3, 2)$

2 marks

1 mark

Q.3) i) $y = \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \dots$ where $0 < x < 3$

Solution:

$$y = \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \dots$$

$$\text{here } a_1 = \frac{x}{3}, \quad r = \frac{x}{3}$$

$$y = \frac{a_1}{1-r} \quad S_\infty = \frac{a_1}{1-r}$$

$$y = \frac{\frac{x}{3}}{1-\frac{x}{3}} = \frac{x}{3} \div \left(1 - \frac{x}{3}\right)$$

$$y = \frac{x}{3} \div \left(\frac{3-x}{3}\right) = \frac{x}{3} \times \left(\frac{3}{3-x}\right) = \frac{x}{3-x}$$

$$y = \frac{x}{3-x}$$

$$y(3-x) = x$$

$$3y - xy = x$$

$$3y = x + xy$$

$$3y = x(1+y)$$

$$x = \frac{3y}{1+y}$$

2 marks

2 marks

1 mark

ii) Maximize $f(x,y) = 2x + y$ subject to the constraints $x + y \leq 6, x + y \geq 1, x, y \geq 0$

Solution:

$$f(x,y) = 2x + y \rightarrow (a)$$

$$x + y \leq 6, \quad x + y \geq 1, \quad x, y \geq 0$$

$$x + y \leq 6 \rightarrow (i)$$

Associated equation of (i) is

$$x + y = 6$$

intercepts

x	6	0
Y	0	6

Take (0,0) as a test point.

$$0 \leq 6 \text{ True}$$

$$x + y \geq 1 \rightarrow (ii)$$

Associated equation of (ii) is

$$x + y = 1$$

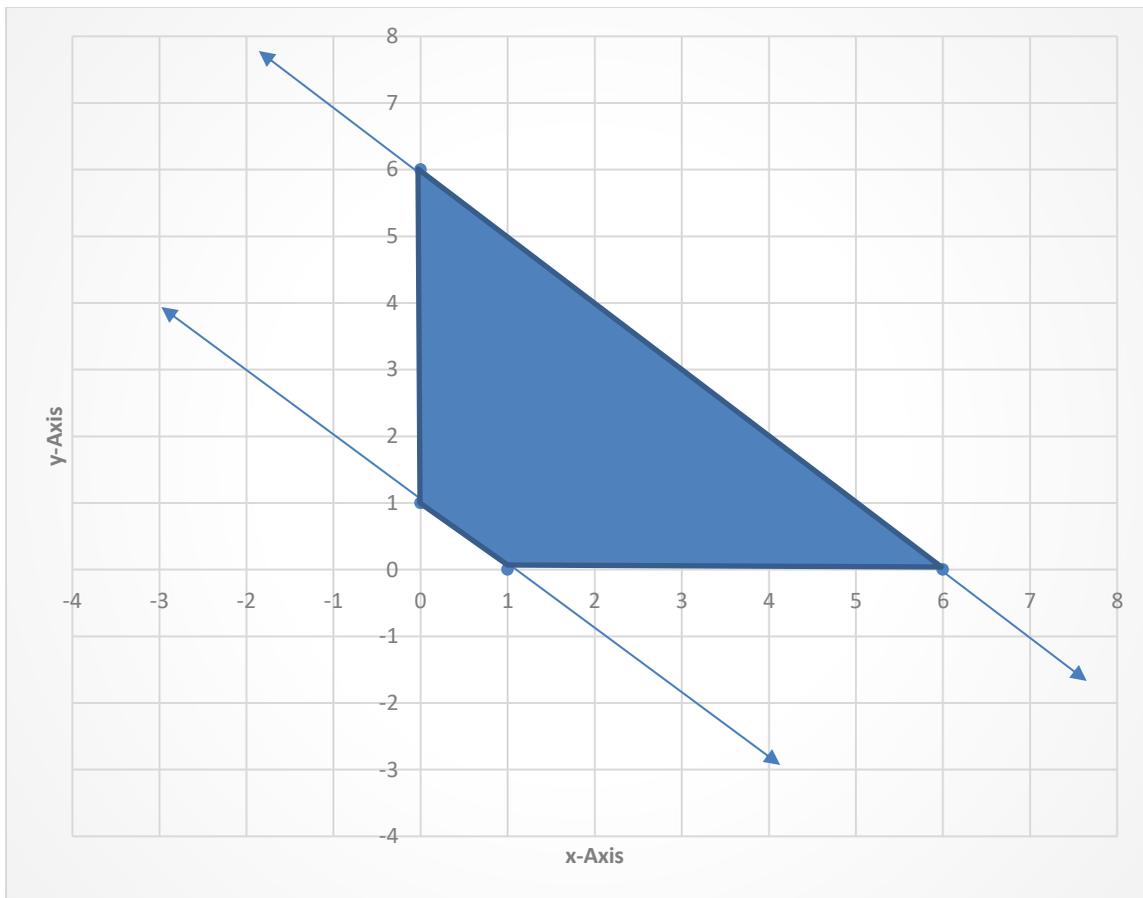
intercepts

x	1	0
y	0	1

Take (0,0) as a test point.

$$0 \geq 1 \text{ false}$$

2 marks



2 marks

So Corners points are (6,0), (0,6), (1,0) and(0,1)

Putting $x = 6$ and $y = 0$ in equation (a).

$$f(6,0) = 2(6) + 0 = 12$$

Putting $x = 0$ and $y = 6$ in equation (a).

$$f(0,6) = 2(0) + 6 = 6$$

Putting $x = 1$ and $y = 0$ in equation (a).

$$f(1,0) = 2(1) + 0 = 2$$

Putting $x = 0$ and $y = 1$ in equation (a).

$$f(0,1) = 2(0) + 1 = 1$$

So $f(x,y)$ is maximum at (6,0).

1 marks

Q.4) i) Find the area of parallelogram whose diagonals are: $\vec{a} = 4\hat{i} + \hat{j} - \hat{k}$,

$$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}.$$

Solution: Given diagonals are $\vec{a} = 4\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -1 \\ 2 & 3 & 4 \end{vmatrix}$$

Expanding from R_1

$$\vec{a} \times \vec{b} = \hat{i}(4 + 3) - \hat{j}(16 + 2) + \hat{k}(12 - 2)$$

$$\vec{a} \times \vec{b} = 7\hat{i} - 18\hat{j} + 10\hat{k}$$

2 marks

For diagonals

$$\text{Area of parallelogram} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\text{Area of parallelogram} = \frac{|7i - 18j + 10k|}{2}$$

$$\text{Area of parallelogram} = \frac{\sqrt{49 + 324 + 100}}{2}$$

$$\text{Area of parallelogram} = \frac{\sqrt{473}}{2}$$

2 marks

1 mark

ii) If $z_1 = 1 + i$, $z_2 = 1 - i$, then find $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right|$

solution:

$$\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right| = \left| \frac{1+i+1-i+1}{1+i-1+i+1} \right|$$

$$= \left| \frac{3}{1+i} \right|$$

$$= \left| \frac{3(1-i)}{(1+i)(1-i)} \right|$$

$$= \left| \frac{3-3i}{1^2-i^2} \right|$$

$$\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right| = \left| \frac{3}{2} - \frac{3}{2}i \right|$$

$$= \sqrt{\frac{3^2}{2} + \frac{3^2}{2}}$$

$$= \sqrt{\frac{9}{4} + \frac{9}{4}}$$

$$= \sqrt{\frac{18}{4}}$$

Therefore $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right| = \frac{3\sqrt{2}}{2} = \frac{3}{\sqrt{2}}$

2 marks

2 marks

1 marks

Q.5) i) How many number each lying between 10 and 1000 can be formed with digits 2,3,4,0,8,9 using only once.

Solution:

Two digits numbers:

$$\text{Total number} = E_1 \cdot E_2 = 5 \times 5 = 25$$

Three digits numbers:

$$\text{Total number} = E_1 \cdot E_2 \cdot E_3 = 5 \times 5 \times 4 = 100$$

Therefore

$$\text{Total} = 25 + 100 = 125$$

2 marks

2 marks

1 marks

ii) Find the maximum and minimum of the function $y = \frac{1}{18-5 \sin(3\theta-45)}$

Solution:

$$\text{Consider } y' = 18 - 5 \sin(3\theta - 45)$$

$$\text{Here } a = 18, \text{ and } b = -5$$

$$M = \text{maximum} = a + |b| = 18 + |-5| = 18 + 5 = 23$$

$$m = \text{minimum} = a - |b| = 18 - |-5| = 18 - 5 = 13$$

$$\text{Now maximum of } y \text{ is } M' = \frac{1}{m} = \frac{1}{13}$$

$$\text{Now minimum of } y \text{ is } m' = \frac{1}{M} = \frac{1}{23}$$

1 mark

2 marks

2 marks